

# Wyoming Department of Education Required Virtual Education Course Syllabus

## BIG HORN COUNTY SCHOOL DISTRICT #1

Program Name	WYCA	Content Area	Mathematics
Course ID	CAMA77684	Grade Level	9, 10, 11, 12
Course Name	AP Calculus BC B	# of Credits	0.5
SCED Code	02125E0.5022	Curriculum Type	Connections Academy

### COURSE DESCRIPTION

*AP Calculus BC, semester B is an extension of Calculus AB. Comparable to college and university calculus, this course will help prepare students for the Calculus BC Advanced Placement exam. The course emphasizes broad concepts and applicable methods. In the second half of the course, the student will explore differential equations, employ analytic geometry to interpret polar, vector, parametric and planar curves, and study polynomial approximations and series. The course provides opportunities for the student to apply concepts to real-world situations.*

### WYOMING CONTENT AND PERFORMANCE STANDARDS

STANDARD#	BENCHMARK
N.RN.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $[5^{\frac{1}{3}}]^3 = 5^{[(\frac{1}{3}) \times 3]}$ to hold, so $[5^{\frac{1}{3}}]^3$ must equal 5.
N.RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
N.RN.3	Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.*
N.Q.2	Define appropriate quantities for the purpose of descriptive modeling.*
N.Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*
N.CN.1	Know there is a complex number $i$ such that $i^2 = -1$ , and every complex number has the form $a + bi$ with $a$ and $b$ real.
N.CN.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
N.CN.3	(+)Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
N.CN.4	(+)Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
N.CN.5	(+)Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument $120^\circ$ .
N.CN.6	(+)Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
N.CN.7	Solve quadratic equations with real coefficients that have complex solutions.
N.CN.8	(+)Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$ .
N.CN.9	(+)Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
N.VM.1	(+)Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$ , $ \mathbf{v} $ , $  \mathbf{v}  $ , $v$ (not bold)).
N.VM.2	(+)Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
N.VM.3	(+)Solve problems involving velocity and other quantities that can be represented by vectors.
N.VM.4	(+)Add and subtract vectors.
N.VM.4a	(+)Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
N.VM.4b	(+)Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
N.VM.4c	(+)Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$ , where $(-\mathbf{w})$ is the additive inverse of $\mathbf{w}$ , with the same magnitude as $\mathbf{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
N.VM.5	(+)Multiply a vector by a scalar.
N.VM.5a	(+)Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(\mathbf{v}_{\text{sub } x}, \mathbf{v}_{\text{sub } y}) = (c\mathbf{v}_{\text{sub } x}, c\mathbf{v}_{\text{sub } y})$ .
N.VM.5b	(+)Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $  c\mathbf{v}   =  c  \mathbf{v} $ . Compute the direction of $c\mathbf{v}$ knowing that when $ c  \neq 0$ , the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$ ) or against $\mathbf{v}$ (for $c < 0$ ).
N.VM.6	(+)Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

N.VM.7	(+)Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
N.VM.8	(+)Add, subtract, and multiply matrices of appropriate dimensions.
N.VM.9	(+)Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
N.VM.10	(+)Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N.VM.11	(+)Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
N.VM.12	(+)Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
A.SSE.1	Interpret expressions that represent a quantity in terms of its context.*
A.SSE.1a	Interpret parts of an expression, such as terms, factors, and coefficients.*
A.SSE.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of $P$ and a factor not depending on $P$ .*
A.SSE.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ .
A.SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
A.SSE.3a	Factor a quadratic expression to reveal the zeros of the function it defines.*
A.SSE.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.*
A.SSE.3c	Use the properties of exponents to transform expressions for exponential functions. For example the expression $1.15^t$ can be rewritten as $[1.15^{(1/12)}]^{(12t)} \approx 1.012^{(12t)}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
A.SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.*
A.APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$ , the remainder on division by $x - a$ is $p(a)$ , so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ .
A.APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.4	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
A.APR.5	(+)Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$ , where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal's Triangle. <sup>1</sup>
A.APR.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
A.APR.7	(+)Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
A.CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
A.CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
A.CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance $R$ .*
A.REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
A.REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A.REI.4	Solve quadratic equations in one variable.
A.REI.4a	Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
A.REI.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$ .
A.REI.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
A.REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
A.REI.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$ .
A.REI.8	(+)Represent a system of linear equations as a single matrix equation in a vector variable.
A.REI.9	(+)Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension $3 \times 3$ or greater).
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
A.REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .
F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$ , $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$ ( $n$ is greater than or equal to 1).
F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
F.IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.*
F.IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
F.IF.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.*
F.IF.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
F.IF.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*
F.IF.7d	(+)Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.*
F.IF.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*

F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F.IF.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
F.IF.8b	Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$ , $y = (0.97)^t$ , $y = (1.01)^{12t}$ , $y = (1.2)^{t/10}$ , and classify them as representing exponential growth and decay.
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
F.BF.1	Write a function that describes a relationship between two quantities.*
F.BF.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
F.BF.1b	Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
F.BF.1c	(+)Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
F.BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*
F.BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
F.BF.4	Find inverse functions.
F.BF.4a	Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse. For example, $f(x) = 2(x^3)$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$ ( $x$ not equal to 1).
F.BF.4b	(+)Verify by composition that one function is the inverse of another.
F.BF.4c	(+)Read values of an inverse function from a graph or a table, given that the function has an inverse.
F.BF.4d	(+)Produce an invertible function from a non-invertible function by restricting the domain.
F.BF.5	(+)Build new functions from existing functions. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
F.LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.*
F.LE.1a	Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.*
F.LE.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.*
F.LE.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*
F.LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
F.LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*
F.LE.4	For exponential models, express as a logarithm the solution to $ab^{(ct)} = d$ where $a$ , $c$ , and $d$ are numbers and the base $b$ is 2, 10, or $e$ ; evaluate the logarithm using technology.*
F.LE.5	Interpret the parameters in a linear or exponential function in terms of a context.*
F.TF.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F.TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F.TF.3	(+)Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$ , $\pi/4$ and $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$ , $\pi + x$ , and $2\pi - x$ in terms of their values for $x$ , where $x$ is any real number.
F.TF.4	(+)Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
F.TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*
F.TF.6	(+)Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
F.TF.7	(+)Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*

F.TF.8	Prove the Pythagorean identity $(\sin A)^2 + (\cos A)^2 = 1$ and use it to find $\sin A$ , $\cos A$ , or $\tan A$ , given $\sin A$ , $\cos A$ , or $\tan A$ , and the quadrant of the angle.
F.TF.9	(+)Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
G.CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
G.CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
G.CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
G.CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
G.CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
G.CO.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
G.CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
G.CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
G.CO.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
G.CO.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
G.CO.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
G.CO.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
G.CO.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
G.SRT.1	Understand similarity in terms of similarity transformations. Verify experimentally the properties of dilations given by a center and a scale factor: <b>a.</b> A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. <b>b.</b> The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
G.SRT.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
G.SRT.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
G.SRT.4	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
G.SRT.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
G.SRT.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
G.SRT.7	Explain and use the relationship between the sine and cosine of complementary angles.
G.SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
G.SRT.9	(+)Derive the formula $A = (1/2)ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.10	(+)Prove the Laws of Sines and Cosines and use them to solve problems.
G.SRT.11	(+)Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

G.C.1	Prove that all circles are similar.
G.C.2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
G.C.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
G.C.4	(+)Construct a tangent line from a point outside a given circle to the circle.
G.C.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
G.GPE.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
G.GPE.2	Derive the equation of a parabola given a focus and directrix.
G.GPE.3	(+)Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
G.GPE.4	For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$ .
G.GPE.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
G.GPE.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
G.GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*
G.GMD.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
G.GMD.2	(+)Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
G.GMD.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*
G.GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
G.MG.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
G.MG.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*
G.MG.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*
S.ID.6a	Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
S.ID.6b	Informally assess the fit of a function by plotting and analyzing residuals.*
S.ID.6c	Fit a linear function for a scatter plot that suggests a linear association.*
S.ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*
S.ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.*

**SCOPE AND SEQUENCE**

UNIT OUTLINE	STANDARD#	OUTCOMES
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<p><b>Unit 1: Differential Equations and More Riemann Sums</b>          In this unit, you are introduced to differential equations and their applications. You will also learn about slope fields and how to use approximation techniques to evaluate definite integrals.</p>		<ul style="list-style-type: none"> <li>•evaluate constants of integration given an initial condition</li> <li>•solve separable differential equations</li> <li>•model various applications using separable differential equations, with particular focus on the study of the equation <math>y' = ky</math> and exponential growth</li> <li>•draw a slope field (or direction field) for a differential equation</li> <li>•interpret a slope field when given it</li> <li>•interpret the solution curve attached to an initial value</li> <li>•use numerical approximation techniques to evaluate definite integrals, where appropriate, including the area under a curve</li> <li>•use Riemann Sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions through algebraic, graphical, and tabular representation (table of values)</li> <li>•discuss error implications of different methods</li> </ul>
<p><b>Unit 2: Supplemental Topics</b>          In this unit, you will bring all of your knowledge together and apply it to new topics. You will learn about the relationships between graphs, relationships between functions, and how to work with improper integrals.</p>		<ul style="list-style-type: none"> <li>•describe and explore the relationship and/or characteristics between the graphs of <math>f</math>, <math>f'</math>, and <math>f''</math>.</li> <li>•compare and contrast the relative growth rate functions, including polynomial, exponential and logarithmic growth.</li> <li>•use integrals to define functions</li> <li>•explore the relationship of functions defined by integrals</li> <li>•explore accumulated change from a rate of change as an application of integrals</li> <li>•use Riemann Sums and trapezoidal sums to approximate definite integrals of functions represented by tables of values</li> <li>•explore functions defined by integrals</li> </ul>
<p><b>Unit 3: Analytic Geometry</b>          In this unit, you will learn about curves defined in ways other than rectangular coordinates. You will learn about parametric, polar, and vector curves. You will wrap up the unit by finding the length of the curves and the area under the curves.</p>		<ul style="list-style-type: none"> <li>•analyze planar curves given in parametric form</li> <li>•compute derivatives of parametric functions and their applications such as finding tangent lines</li> <li>•find the arc length of a curve described in Cartesian form as well as parametric form</li> <li>•convert between parametric form and other forms (such as rectangular/Cartesian form)</li> <li>•graph curves in parametric form, including using technology to do so</li> </ul>

**Unit 4: Series and Convergence**

In this unit, you will investigate series, including arithmetic, geometric, alternating, and harmonic series. You will also learn about the relationship between series and integrals.

- review familiar series including arithmetic and geometric series
- compute, using familiar series, partial sums as well as infinite sums if they exist
- evaluate if a familiar series is convergent—particularly using geometric series
- find the general term of a series and identify whether it is arithmetic, geometric or neither
- use graphs and technology to explore convergence and divergence
- determine whether a sequence converges (to a finite value) or diverges
- explore examples that involve the application of geometric series including repeating decimal expansion
- view terms of series as areas of rectangles and their relationship to integrals
- identify common series such as alternating series and the harmonic series
- explore and compute the error bound of alternating series

**Unit 5: Polynomial Series and Approximations**

In this unit, you will continue to learn about series, including Maclaurin, Taylor, power, and polynomial series. You will use your new series to model real-life behavior.

- calculate the nth order Maclaurin polynomial approximation for a given function
- be familiar with the Maclaurin series for the functions  $e^x$ ,  $\sin x$ ,  $\cos x$ , one over the quantity one minus  $x$
- approximate a function with an appropriate Maclaurin series
- calculate the nth order Taylor polynomial approximation for a given function centered at  $x = a$
- approximate a function or value (such as  $\ln 3$ ) with an appropriate Taylor series
- explore and compute the Lagrange error bound for Taylor polynomial approximations to functions with graphical demonstration of convergence, including use of appropriate technology to do so
- use Taylor approximations in real-life applications such as modeling and calculating differential equations
- explore functions defined by power series
- be able to find the power series of a given function by formally manipulating (and computing with) known power series
- use techniques such as substitution differentiation and integration with the power series of a given function

	<ul style="list-style-type: none"> <li>•calculate the interval and radius of convergence of a given power series</li> <li>•be familiar with the interval of convergence for the common power series (Maclaurin series) for <math>e^x</math>, <math>\sin x</math>, <math>\cos x</math>, one over the quantity one minus <math>x</math></li> <li>•use polynomial series to approximate irrational numbers</li> <li>•use polynomial series to solve differential equations</li> <li>•use polynomial series to Do rectilinear motion</li> <li>•use polynomial series to model real-life behavior</li> <li>•use polynomial series to evaluate limits</li> </ul>
<p><b>Unit 6: Getting Ready for the Exam</b>  In this unit, you will learn about the structure of the AP Exam and will practice exam questions to prepare for the exam.</p>	<ul style="list-style-type: none"> <li>•understand the format of the AP Exam</li> <li>•understand test taking techniques for taking the multiple choice part of the exam</li> <li>•understand question types to expect on the multiple choice part of the exam</li> <li>•learn which calculator models may be used on the exam</li> <li>•understand which calculator operations are permitted</li> <li>•understand the degree of accuracy required on the exam</li> <li>•understand the format of free response questions on the exam</li> <li>•review a list of topics frequently tested in the FRQ section</li> <li>•review a list of common mistakes made on the exam</li> <li>•understand how the BC score is computed</li> <li>•understand how the AB sub-score is computed</li> </ul>