

# Wyoming Department of Education Required Virtual Education Course Syllabus

## Sheridan County School District # 1

Program Name	Sheridan County School District #1 Virtual School	Content Area	MA
Course ID	AC02064	Grade Level	11
Course Name	Integrated Mathematics III-CCSS	# of Credits	1
SCED Code	02064	Curriculum Type	Acellus

### COURSE DESCRIPTION

Acellus Integrated Mathematics III is the last course of a three-part series that includes algebra, geometry, probability, and statistics. This high school math pathway is patterned after an approach typically seen internationally. Acellus Integrated Mathematics III is A-G Approved through the University of California.

STANDARD #	BENCHMARK <a href="#">(Standard/Indicator) Use the Standards and Benchmarks as Spreadsheets</a>
N.CN.8	(+)Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$ .
N.CN.9	(+)Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
A.SSE.1	Interpret expressions that represent a quantity in terms of its context.*
A.SSE.1a	Interpret parts of an expression, such as terms, factors, and coefficients.*
A.SSE.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
A.SSE.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ .
A.SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.*
A.APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$ , the remainder on division by $x - a$ is $p(a)$ , so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ .
A.APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.4	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

A.APR.5	(+)Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$ , where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal's Triangle. <sup>1</sup>
A.APR.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
A.APR.7	(+)Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
A.CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
A.CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
A.CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance $R$ .*
A.REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
A.REI.11	Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
F.IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.*
F.IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

F.IF.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
F.IF.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*
F.IF.7d	(+)Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.*
F.IF.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
F.BF.1	Write a function that describes a relationship between two quantities.*
F.BF.1b	Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
F.BF.1c	(+)Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
F.BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
F.BF.4	Find inverse functions.
F.BF.4a	Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse. For example, $f(x) = 2(x^3)$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$ ( $x$ not equal to 1).
F.BF.4c	(+)Read values of an inverse function from a graph or a table, given that the function has an inverse.
F.LE.4	For exponential models, express as a logarithm the solution to $ab^{(ct)} = d$ where $a$ , $c$ , and $d$ are numbers and the base $b$ is 2, 10, or $e$ ; evaluate the logarithm using technology.*

F.TF.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F.TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F.TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*
G.SRT.9	(+)Derive the formula $A = (1/2)ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.10	(+)Prove the Laws of Sines and Cosines and use them to solve problems.
G.SRT.11	(+)Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
G.GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
G.MG.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
G.MG.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*
G.MG.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*
S.ID.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*
S.IC.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.*
S.IC.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*
S.IC.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.*

S.IC.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*
S.IC.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.*
S.IC.6	Evaluate reports based on data.*
S.MD.6	(+)Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).*
S.MD.7	(+)Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).*

**SCOPE AND SEQUENCE**

UNIT OUTLINE	STANDARD#	OUTCOMES OBJECTIVES/STUDENT CENTERED GOALS
Unit 1 – Algebraic Expressions	A.SSE.1a, A.SSE.1b, A.CED.1, A.CED.4	In this unit, students are introduced to the course and given an overview of the course objectives, including the use of algebraic expressions, graphs, and tables. Real numbers are reviewed and classified into natural and whole numbers, integers, rational, and irrational numbers. Students practice simplifying algebraic expressions and evaluating expressions using a given value in place of the variable. Students are reintroduced to solving simple algebraic equations and given several examples. The basic rules for manipulating an equation are also covered. Students practice solving literal equations for any variable. The differences between equations and inequalities are discussed and students extend their understanding of inequalities to compound inequalities. The concept of absolute value is presented and several examples are given to demonstrate its use in equations and inequalities.

Unit 2 – Functions	A.SSE.1, A.SSE.1a, A.SSE.2, F.IF.7, F.IF.7b, A.CED.2, A.CED.3, F.IF.4, F.IF.6, F.IF.8, F.IF.9, F.BF.1, F.BF.3	This unit works to deepen students' understanding of functions and the important part that functions play in mathematics and in the real world. Students begin by studying what a function is and reviewing slope-intercept and point-slope equations of a line. Then they move onto piece-wise defined functions and are expected to correctly utilize piece-wise functions provided by properly determining which formula to use. Students also study the step function and find specific values on the step function. Students use higher order thinking skills to read tables and graphs and solve step function problems by finding values. Terminology relating to correlation is introduced, and students learn how to discover a trend in real-world data using a linear model and finding the equation. Students analyze the difference between the graph of a line and the graph of a linear inequality. Students learn what even and odd functions are and classify functions as either even, odd, or neither.
Unit 3 – Linear Systems	A.REI.11, A.CED.2, A.CED.3	In this unit linear systems are explored. Students study both systems of linear equations and systems of linear inequalities. They expand on the concept of systems of inequalities and demonstrate their usefulness in solving business problems presented as word problems. Systems of equations containing three variables are also introduced. The student will be shown the solution to such a system using a graphing calculator, and the elimination method will be applied to find the solution. Finally, students are shown how to represent systems of equations using matrices, and how to use this information to solve systems of equations easily using matrices in a graphing calculator.
Unit 4 – Quadratic Functions	N.CN.8, A.SSE.1a, A.SSE.2, A.APR.3, F.IF.7, A.CED.1, A.CED.2, A.CED.3, F.IF.4, F.IF.5, F.IF.6, F.IF.7, F.IF.8, F.IF.9, F.BF.1	At the start of this unit, parabolic functions are introduced and the student is shown how to apply various transformations to affect the graph. Students then study the standard form for quadratic functions and learn how to easily find the vertex of a parabola when provided an equation in standard form. Understanding is established to fit a parabola to a set of three points using a system of equations and matrices - this technique is then compared to quadratic regression. The concept of factoring quadratic equations is introduced, and students are shown how to find a parabola's x-intercepts by factoring the quadratic's equation. Students study and use the quadratic formula. Students learn how to solve quadratic inequalities where the quadratic is factorable. The effects of the inequality on the graph of the function are also discussed. Students learn of the origin, use, and format of complex numbers, including when complex numbers occur while working with quadratic equations and complex number arithmetic. Students are taught how to graph complex numbers and find the distance between two complex numbers using the Pythagorean Theorem. The effects of raising complex numbers to various powers is covered, and also the addition of complex numbers and vectors.

Unit 5 – Polynomials	N.CN.8, N.CN.9, A.SSE.1, A.SSE.1a, A.SSE.2, A.APR.1, A.APR.2, A.APR.3, A.APR.5, A.APR.6, F.IF.7, F.IF.7c, F.IF.4, F.IF.5, F.IF.6, F.BF.1	This unit helps students to gain a strong understanding of polynomial functions. Students learn how to factor a polynomial and convert basic polynomial functions between factored and non-factored form. More complicated polynomial equations are factored through factoring by grouping. Students study polynomial division problems. Synthetic division is described and its limitations are discussed. Students learn to use the Rational Root Theorem and synthetic division to find the roots of a polynomial equation. Students learn the Fundamental Theorem of Algebra and use it to determine the number of expected roots for a polynomial equation, as well as a valid grouping of complex numbers. Students learn the method for finding the y-intercept of a polynomial function. Several functions are graphed with a calculator and by hand.
Unit 6 – Radical Functions	A.REI.2, F.IF.7, F.IF.7b, F.IF.8, A.CED.4, F.BF.1b, F.BF.4, F.BF.4a, F.BF.1c, F.BF.4c	This unit introduces radical functions. At the start of the unit, we expand on the properties of exponents covered earlier to include fractional exponents. The basic concept of adding and simplifying expressions involving fractional exponents is covered and several examples are provided. Students then learn how to properly solve more advanced radical equations, including examples showing how to convert radical equations into a polynomial equations. The different ways to combine functions are discussed, and composition is introduced. Inverse functions are introduced and studied. Students will observe the graphs of the basic radical functions and examples will be provided where transformations are applied to these functions.
Unit 7 – Exponential and Logarithmic Functions	A.SSE.1b, A.REI.11, F.IF.7, F.IF.7e, F.IF.8, A.CED.2, F.BF.1, F.BF.4a, F.LE.4	This unit focuses on exponential and logarithmic functions. Students will learn to convert exponential functions back and forth between exponential form and logarithmic form. Students will also learn how to evaluate logarithmic functions and will understand and implement properties of logarithms. Students will be required to find the exact value of some logarithmic problems without the use of a calculator. Students will prove the big three laws of logarithms. They will learn how to prove the formula that allows the change from logs with one base to logs with another. Students will then use that knowledge to solve problems. Students will solve exponential and logarithmic equations in mathematical and real-world application problems. They will calculate compound interest in real-world applications.

Unit 8 – Rational Functions	A.SSE.1, A.SSE.1a, A.SSE.1b, A.SSE.2, A.APR.1, A.APR.3, A.APR.6, A.APR.7, A.REI.2, A.REI.11, F.IF.7, F.IF.7d, A.CED.1, A.CED.2, F.IF.4, F.IF.5, F.BF.1, F.BF.3	In this unit, students will study and learn how to graph rational functions. They will identify vertical asymptotes, horizontal asymptotes, y-intercepts, and x-intercepts. Students will add, subtract, multiply and divide rational expressions. Students will understand closure with rational expressions and that when they add, subtract, multiply, or divide non-zero rational expressions, they will get a rational expression back. Students will practice solving rational equations, and will be able to identify when a rational equation has no solution. Students will practice solving rational inequalities. Students will learn how to use a graphing calculator to solve equations. Students will understand Number Theory and use it complete various proofs. Students will study the end behavior of linear, quadratic, polynomial, radical, exponential, and rational functions. For each type, they will be required to indicate what happens as x approaches infinity and as x approaches negative infinity.
Unit 9 – Sequences and Series	A.SSE.1a, A.SSE.1b, A.SSE.4, F.BF.1	This unit begins with students analyzing and learning the difference between explicit and recursive definitions. Students will be required to write explicit formulas and recursive definitions. Students will also be required to find a given term in a defined sequence. Next, students will study arithmetic sequences, and be required to find missing terms and future terms. They will study geometric sequences, and be required to find missing terms and future terms when part of the sequence is given. Students will also be expected to find future terms given the first term and common ratio of the geometric sequence. Students will study both arithmetic series and geometric series, and know how to find the sum. They will use critical thinking to apply geometric series to solve real-world application problems.
Unit 10 – Probability and Statistics	S.ID.4, S.IC.1, S.IC.2, S.IC.3, S.IC.4, S.IC.5, S.IC.6, S.MD.6, S.MD.7	This unit strives to help students develop a deeper understanding of probability and statistics. In this unit, students will evaluate permutations and combinations, involving real-world situations. They will study probability and learn to distinguish between independent and dependent events. Students will find probabilities of various events and learn to calculate the probability of multiple events. Students will also learn to calculate conditional probability. Students will study and interpret probability models, ie. by using a calculator to generate random numbers. Students will study and analyze data and given a set of data, they will learn how to find the standard deviation and the variance. Students will study randomized experiments and how they are applicable in real-world contexts. They will also study binomial distributions, normal distributions, and probability distribution functions. Finally, students will find the expected value of various scenarios

Unit 11 – Periodic Functions	F.TF.1, F.TF.2, F.TF.5, F.IF.4, F.IF.7e	This unit begins with students learning the definition of periodic. Given a graph, students are required to determine the period. Next, the Unit Circle is studied and students are given an angle and required to find the coordinates of the point where that angle would intersect the Unit Circle. Students learn how to use radian measure and practice converting degrees to radians. The sine, cosine, and tangent functions are studied. Students learn how to translate trigonometric functions. Given the graph of a trigonometric function, they are required to find its equation. Finally, students study reciprocal trigonometric functions.
Unit 12 – Trig Identities and Equations	A.APR.4, G.SRT.10, G.SRT.11	In this unit, students will study and utilize important trigonometric functions. This unit begins with students learning and applying the basic trig identities to solve problems. Students build upon this understanding and practice solving trig equations. Then students study and practice using inverse trig functions. Students will find missing side lengths of right triangles, given the angle theta and one of the sides. Students will use the Law of Sines, and the Law of Cosines to find missing elements.
Unit 13 – Area	G.SRT.9, G.MG.1	In this unit the area formulae for parallelograms, triangles, and trapezoids are developed by extending the students understanding of the area of a rectangle. Rhombus, kite, and regular polygon areas are developed similarly using the triangle area formula. Students practice using these formulae to find the area of the above mentioned figures. The areas and perimeters of similar polygons are introduced in the context of enlarging a patio allowing students to see how the geometry fits easily in a commonplace situation. The students apply basic trigonometry previously learned to find the area of triangles when critical information is missing.
Unit 14 – Surface Area and Volume	G.GMD.4, G.MG.1, G.MG.2, G.MG.3	The students are challenged to extend their thinking and visualization skills to three dimensions in this unit. They are introduced to the vocabulary of polyhedra and given a short history of Leonard Euler and his formula relating to polyhedra. The students then learn how to find the surface areas and volumes of prisms, cylinders, pyramids, and cones both from diagrams and verbal descriptions. Sphere volume and surface area are discussed. As with area, the students investigate the relationship between surface areas and volumes of similar polyhedral, this time in the context of nutrition. Classroom activities in this unit include extensions of the concepts dealing with Cavalieri's Principle, density, scaling, cross sections and the surface area to volume ratio.